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Event-Based Analysis of Changes in Surfaces

Yukio Sadahiro

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Center for Spatial Information Science
University of Tokyo
7-3-1, Bunkyo-ku, Tokyo 113-8656, Japan

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Yukio Sadahiro

Center for Spatial Information Science and Department of Urban Engineering

University of Tokyo

7-3-1, Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

Phone: +81-3-5841-6273

Fax: +81-3-5841-8521

E-mail: sada@okabe.t.u-tokyo.ac.jp

Event-Based Analysis of Changes in Surfaces

Abstract

This paper develops a method for analyzing changes in a surface, a scalar function defined over a two-dimensional region. The method is based on the topological method for static surfaces that uses critical points (peaks, bottoms, and cols) and their connecting lines. To extend the topological method to spatio-temporal domain, four types of primitive events are proposed: 1) generation, 2) disappearance, 3) movement, and 4) switch. The change of a surface is described by a combination of these primitives. From surfaces of two times a set of primitive events that causes the change is deduced. They are stored in GIS as spatial objects with attributes, and their spatio-temporal pattern is visually analyzed. To test the validity of the method, the change of a retail cluster in Shinjuku and Shibuya area in Tokyo is analyzed. The empirical study yielded some interesting findings that help us understand changes in the spatial structure of retailing.

1. INTRODUCTION

Surface, a scalar function defined over a two-dimensional region, is one of the most important objects in GIS. Continuous variables such as terrain elevation and atmospheric temperature are approximately modelled by TIN or grid in GIS. Surfaces are also generated by conversion of other spatial objects; point distributions are aggregated into some spatial units and smoothed to represent the density distribution of points.

Scalar fields modelled as surfaces in GIS are analyzed not only visually but also mathematically and even statistically. To this end there have been proposed numerous methods in the literature, and they can be classified into three types as follows. The first type of method is so called geostatistics including variogram, correlogram, and kriging (Isaaks and Srivastava 1989; Cressie 1993). It is used for spatial analysis as well as spatial interpolation, and plays a major role in surface analysis. For instance, variogram cloud and its related concepts such as sill and range are useful for describing spatial characteristics of surfaces. The second type of method focuses on modelling surfaces by mathematical functions. It includes trend surface analysis (Tobler 1964; Bailey and Gatrell 1995) and its extensions (Hagget 1968; Griffith 1981). Trend surface analysis is suitable for modelling simple surfaces, say, a uncentered population distribution. However, since they assume the form of surface functions a priori, operational difficulties arise in modelling multicentered distributions. The third type of method deals with the topological structure of surfaces, so they can be called topological or qualitative method. Warntz (1966), Warntz and Waters (1975), and Okabe and Masuda (1984) extract a graph consisting of peaks, bottoms, cols, and their connecting links from a surface, in order to describe its global spatial structure. Topological method is characterized by its simplicity; we can easily understand the rough form of a surface by the graph.

These methods are basically developed for analyzing static aspects of surfaces, that is, the state of surfaces at a fixed time. However, with the advance of GIS technology, demand for dynamic analysis of surfaces is now rapidly growing. Spatial data acquisition tools such as GPS and RS now provide us a large amount of spatio-temporal data of surfaces, and we can easily see the change of surfaces over time through GIS. One method to analyze the change is to apply the existing methods to snapshot data and discuss the results on the temporal axis. Unfortunately, since the existing methods do not consider the dynamics of surfaces explicitly, they often fail to detect even basic transformations of surfaces such as translation and rotation. In spatio-temporal analysis it is obviously necessary to take dynamics into account explicitly in its methodology.

To answer this demand, the present paper proposes a new method for analyzing the change of surfaces. The method is based on the topological method mentioned above, that

is, its extension to spatio-temporal domain. Following Okabe and Masuda (1984), we first outline the topological method in section 2. To extend their method, we then propose four types of local changes in surfaces which we call primitive events. The change of a surface is described by a combination of primitive events; given surfaces of two times, we deduce a set of primitives that causes the change. The primitive events deduced are stored in GIS as spatial objects with attributes, and their spatio-temporal pattern is visually analyzed. In order to test the validity of the method, section 3 empirically analyzes the change of a retail cluster in Shinjuku and Shibuya area in Tokyo, Japan. Section 4 summarizes the conclusions with discussion.

2. EVENT-BASED METHOD FOR ANALYZING CHANGES IN SURFACES

2.1 Topological method for analyzing surfaces

The method we will propose is based on the topological method for analyzing surfaces originally developed by Warntz (1966) and Warntz and Waters (1975), and extended by Okabe and Masuda (1984). We thus outline the topological method before we extend it to spatio-temporal domain.

Suppose a continuous function $f(\mathbf{x})$ defined over a connected region S , which is usually assumed to be smooth and differentiable at any point in S . Approximating the local surface around a point in S by a quadratic function, we can classify every point in S into one of the four types of points: 1) peak, 2) bottom, 3) col, and 4) slope (Figure 1). The first three points are called *critical points* (Iri *et al.*, 2000). The Poincaré index theorem assures

$$n_p + n_b - n_c = 1, \quad (1)$$

where n_p , n_b , and n_c are the numbers of peaks, bottoms, and cols, respectively.

FIG. 1. Four types of local surfaces.

The integral curves of the steepest ascent vectors of $f(\mathbf{x})$ compose a set of curves connecting peaks and bottoms. Some curves pass through cols; at each col four integral curves meet. We call these curves *corridors* hereafter. Critical points and corridors provide a graph representation of a surface which we call the *structural graph*, as shown in Figure 2b. The graph shows the global structure of a surface, so that we can easily understand its rough form. Okabe and Masuda (1984) uses this representation of surfaces to compare population distributions of various cities in Japan.

FIG. 2. Structural graph of a surface. a) Original surface, b) critical points and corridors.

The topological method detects all peaks and bottoms no matter how their height or depth is small. In spatial analysis, however, only significant critical points are of interest. To remove insignificant peaks and bottoms, a threshold value is often used for defining peaks and bottoms. For example, the height of a peak is defined as its relative height compared to that of its connecting cols, and peaks whose height is smaller than a threshold value are neglected.

2.2 Primitive events

To extend the topological method outlined above, we use the concept of primitive events defined for critical points and corridors (Sadahiro and Umemura, 2000). Suppose a spatio-temporal function $f(\mathbf{x}, t)$ defined over a connected region S for a certain time period, which is smooth and differentiable with respect to \mathbf{x} at any point in S .

We first define primitive events for peaks. *Generation* of a peak is defined as the change of a point on $f(\mathbf{x}, t)$ from a slope to a peak which is caused by the rise of the point (Figure 3a) or the fall of its surrounding surface. In contrast, *disappearance* of a peak is the change from a peak to a slope (Figure 3b). *Movement* of a peak is defined as the continuous locational change of a peak which is caused by the smooth change of a surface (Figure 3c). Though spatial objects that compose a surface may not actually move, it is convenient in spatial analysis to represent such changes as if peaks actually move.

FIG. 3. Three types of primitive events for peaks. a) Generation, b) disappearance, and c) movement.

The three primitive events can be classified further by considering the change of $f(\mathbf{x}, t)$ when the event occurs. As mentioned above, peaks are generated by either the rise of a slope point or the fall of its surrounding surface. We distinguish these changes using the derivative of $f(\mathbf{x}, t)$ with respect to t :

$$f'(\mathbf{x}, t) = \frac{\partial}{\partial t} f(\mathbf{x}, t). \quad (2)$$

Generation+, abbreviated as G+, is the generation of a peak when $f'(\mathbf{x}, t) > 0$, that is, generation by the rise of a slope point. Similarly, *generation-* (G-) is the generation of a peak when $f'(\mathbf{x}, t) \leq 0$. Disappearances are also classified further into two types: D+ ($f'(\mathbf{x}, t) \geq 0$) and D- ($f'(\mathbf{x}, t) < 0$). The former indicates the disappearance of a peak by the rise its surrounding surface, not by its own fall. Movements are classified into three types: M+,

M_- , and M_0 . If a peak moves and its height grows, the movement is denoted by M_+ . When a peak moves keeping its height, the movement is denoted by M_0 .

Generation and disappearance of peaks are described in relation to their neighboring peaks. As shown in Figure 2, each col has its two neighboring peaks. This and equation (1) assure that generation of a peak is always accompanied by generation of its neighboring col, and that the col has another neighboring peak (Figure 4). To state this explicitly, in Figure 4, we say that the peak B was generated from the peak A which is connected to the newly generated col C. This representation is reasonable because the peak B belonged to the slope of A before its generation; it looks as if the mountain represented by the peak A born its child mountain represented by the peak B. Similarly, for disappearance of peaks, we say that a peak was absorbed into the peak connected to the vanished col. This is because disappearance of a peak reflects that the mountain represented by the peak is combined into its neighboring mountain.

FIG. 4. Generation of a peak. We say "the peak B was generated from the peak A."

Generation, disappearance, and movement are defined for bottoms and cols as well as peaks. Generation and disappearance of a bottom are again related to its neighboring bottom. Generation and disappearance of a col is accompanied by those of a peak or a bottom, so that they are described together. For example, we say that a col vanished with the disappearance of its neighboring peak.

Primitive events for corridors are somewhat richer than those for critical points. Generation and disappearance are defined in connection with those of peaks and bottoms; they are always accompanied by those of peaks or bottoms. Movement, which does not change the topology of the structural graph, is also defined for corridors. In addition to these events, an event which we call *switch* can occur on corridors. Every corridor has a col on one end, and a peak or a bottom on the other. Keeping the connection with a col, a corridor can change the critical point on the other end (Figure 5).

FIG. 5. Switch event.

Among primitive events defined above, generation, disappearance and switch are called *topological events* in this paper, since they change the topology of the structural graph. In contrast, movement is called *non-topological event*.

Any change of the structural graph can be classified into one or a set of primitive events. This implies that the structural change of a surface can be described by the primitive events, which provides a means of visualizing and analyzing changes in

surfaces.

2.3 Significance of critical points

As well as the static topological method, our extension to spatio-temporal analysis is concerned with the significance of critical points. To extract significant critical points, we define the relative height and depth of a col at \mathbf{x} as

$$h(\mathbf{x}) = f(\mathbf{x}) - \max\{f(\mathbf{x}_{B1}), f(\mathbf{x}_{B2})\} \quad (3)$$

and

$$d(\mathbf{x}) = \min\{f(\mathbf{x}_{P1}), f(\mathbf{x}_{P2})\} - f(\mathbf{x}), \quad (4)$$

respectively, where \mathbf{x}_{Pi} and \mathbf{x}_{Bi} are the locational vectors of peaks and bottoms directly connected with the col, respectively. If these values are very small, we find the col is not significant. Formally, a col at \mathbf{x} is significant if

$$h(\mathbf{x}) \geq T \quad (5)$$

and

$$d(\mathbf{x}) \geq T, \quad (6)$$

where T is a threshold value that is given extrinsically. Insignificant cols are removed from the structural graph with its connecting corridors and neighboring peaks or bottoms if necessary.

The choice of the threshold value T depends on the circumstances. One method is to adopt a value around which the resultant structural graph is stable for a wide range of T (detailed discussion can be found in Okabe and Masuda 1984 and Okabe and Masuyama 2000). Otherwise, we can choose a value which is suitable for the purpose of analysis. If we are interested in the global structure of a surface, we should adopt a large value for T . If we want to analyze details of a surface, a small value or even zero value is desirable.

2.4 Deduction of primitive events

Change of a surface is mathematically defined as the change of a continuous function from $f(\mathbf{x}, t_1)$ to $f(\mathbf{x}, t_2)$. If the set of all primitive events between t_1 and t_2 is explicitly recorded in GIS data, we can easily visualize and analyze them, which helps us understand the spatio-temporal structure of the surface $f(\mathbf{x}, t)$. However, in most cases, we can obtain only the snapshots of $f(\mathbf{x}, t)$, for example $f(\mathbf{x}, t_1)$ and $f(\mathbf{x}, t_2)$. We thus have to deduce primitives events between t_1 and t_2 from $f(\mathbf{x}, t_1)$ and $f(\mathbf{x}, t_2)$.

To this end, we first interpolate $f(\mathbf{x}, t)$ from t_1 to t_2 by linear interpolation. The surface function at t is given by

$$f(\mathbf{x}, t) = \frac{t_2 - t}{t_2 - t_1} f(\mathbf{x}, t_1) + \frac{t - t_1}{t_2 - t_1} f(\mathbf{x}, t_2). \quad (7)$$

Though linear interpolation may not be always valid, it is a good approximation of $f(\mathbf{x}, t)$

when no additional information on $f(\mathbf{x}, t)$ is available.

We then deduce primitive events between t_1 and t_2 from $f(\mathbf{x}, t)$. Let us briefly show a procedure for deducing events for peaks. Assume that the surface is represented by a set of scalar values measured at sample locations and that they are spatially interpolated by planes, as frequently done in TIN and grid. We first examine all the sample points and their neighboring points from t_1 to t_2 to know whether they could be critical points during the period, setting temporarily the threshold value T to zero. For candidate points we then check whether they could be significant with respect to the true threshold value T , comparing their height with that of all the candidates of cols. From this we obtain a list of possible generations and disappearances in temporal order. Following the list, we reproduce the surface and its structural graph to examine whether topological events could really occur, and specify all the generations and disappearances of peaks. In contrast to topological events, movement is defined for intervals rather than instants. Therefore, we discretely trace the location of peaks between topological events.

2.5 Analysis of events

Once primitive events are deduced, they can be stored in GIS as spatial objects with attributes. Therefore, one method to analyze the change of a surface is to visualize the distribution of primitive events and the structural graph through GIS. The spatial distribution of events can be visualized for any time interval; the structural graph and its original surface can also be visualized for instants.

Primitive events are defined for critical points and corridors. However, this does not imply that all the primitive events are equally important in any application; analysts should focus on primitive events closely related to their interest. In fact, we often pay attention to the change of peaks and bottoms when analyzing surfaces, since their spatial arrangement characterizes the global spatial structure of surfaces. In addition, as Okabe and Masuda (1984) noted, cols and either peaks or bottoms are redundant descriptors of the structural graph. This is because of two reasons: 1) a col is always directly connected to two peaks and two bottoms; 2) bottoms are always encircled by corridors connecting peaks and cols, and so are peaks by those connecting bottoms and cols. Because of these properties and equation (1), most of the topological events of cols and corridors are accompanied by those of peaks or bottoms. Even if we remove bottoms, cols, and corridors connecting bottoms and cols, we can restore the original graph from the remaining peaks and corridors. Consequently, to focus on either peaks or bottoms is a rational choice.

However, when we analyze the population distribution, not only peaks and bottoms but also cols and corridors are of interest because the doughnut phenomenon is of great

importance in population research. The doughnut phenomenon is represented as a circuit with a col in the structural graph, and its appearance is accompanied by generation of a bottom or switch of a corridor.

Spatio-temporal statistics is also useful when analyzing the spatio-temporal distribution of topological events of critical points. For example, Knox (1964) and Mantel (1967) propose statistical tests for analyzing clusters in spatio-temporal point distributions. Openshaw (1994) develops an exploratory method for detecting space-time-attribute patterns in point distributions. Jacquez (1996) extends the K-function method to spatio-temporal analysis. These methods are useful for analyzing spatio-temporal pattern in the distribution of topological events, especially for examining whether they are clustered in the spatio-temporal dimension.

3. EMPIRICAL STUDY

This section empirically analyzes the change of a surface in order to test the validity of the method proposed in the previous section. We investigate the change of retail activity in Shinjuku and Shibuya area in Tokyo, Japan (Figure 6). The study region contains the biggest retail cluster in Japan, surrounded by urban parks, residential areas and business districts. The Yamanote Line and Meiji Street pass through the region, and there are five major railway stations: Shinjuku, Yoyogi, Harajuku, Shibuya, and Ebisu.

FIG. 6. Shinjuku and Shibuya area in Tokyo, Japan.

Spatial data used in the analysis are based on the list of retail stores and restaurants in the *NTT telephone directory* which is published monthly. We extracted their addresses in September every other year from 1990 to 98 in ASCII format, and converted them into spatial data by geocoding. Table 1 shows the number of retail stores and restaurants in the study region.

TAB. 1. The number of retail stores and restaurants, 1990-98.

From the point data we calculate the density distribution of stores using the following equation:

$$f(\mathbf{x}) = \sum_i \exp \left[- \left(\frac{|\mathbf{x} - \mathbf{y}_i|}{R} \right)^2 \right], \quad (8)$$

where \mathbf{y}_i is the locational vector of store i and R is the distance decay parameter. We adopt two values for R , 100m and 200m, and perform analysis in order to look at the change in both local and global scales. Figure 7 shows the density distribution function in

1998 for $R=200\text{m}$.

FIG. 7. Density distribution function of retail stores and restaurants, 1998 ($R=200\text{m}$).

Figure 7 shows the global structure of the retail cluster. There are two big subclusters, one at the northeast of Shinjuku, and the other at the west of Shibuya. In addition, there are smaller subclusters at the east of Harajuku and the southwest of Ebisu. These four subclusters form a big retail belt running from north to south between the Yamanote Line and Meiji street.

From the density functions we calculate critical points and corridors. For the threshold value T , which are used for extracting significant critical points, we tried various values ranging from 0.01 to 1.0. Here we adopt 0.2 for $R=100\text{m}$ and 0.1 for $R=200\text{m}$, respectively, because the structural graph was relatively stable around these values. The graph is calculated for every other year from 1990 to 98, and primitive events are deduced for each two-year period. In the following discussion we focus on the change of peaks because they represent subcenters of the retail cluster which characterize the basic structure of the cluster.

3.1 Global analysis

We first discuss the change of global structure of the retail cluster using the density function of $R=200\text{m}$. The number of peaks and topological events are shown in Table 2. The structural graphs in 1990 and 1998 are shown in Figure 8.

TAB. 2. The number of peaks and topological events, 90-98 ($R=200\text{m}$).

FIG. 8. The structural graphs of the density function of retail stores and restaurants, 1990 and 98 ($R=200\text{m}$).

In Figure 8 we notice that the graph had been considerably stable during the period; since 1990 only two peaks appeared and one peak disappeared. We should note that topological events shown in Table 2 include two pairs of generations and disappearances (94-96 and 96-98) that occurred at the same peak located at the west of Shinjuku. Two peaks that disappeared until 1992 were absorbed into the peaks at Harajuku and Ebisu which represent big subclusters. From this we may say, at the global scale, that small subclusters were integrated into larger ones during 90-92. However, in general, the spatial structure of the retail cluster had been quite stable from 1990 to 98.

Movement of peaks during 90-98 is shown in Figure 9. The figure indicates that

peaks had generally moved away from railway stations. This is probably because of the following reason. There remains only little space available for new stores and restaurants near stations such as Shinjuku and Shibuya. This promotes expansion of retail clusters from stations to its surrounding area. However, clusters usually expand not uniformly but in some specific directions, which results in movement of peaks.

FIG. 9. Movement of peaks, 90-98 ($R=200\text{m}$).

3.2 Local analysis

Let us then turn to local analysis where $R=100\text{m}$ in equation (8). The number of peaks and topological events are shown in Table 3. The structural graphs in 1990 and 1998 are shown in Figure 10.

TAB. 3. The number of peaks and topological events, 90-98 ($R=100\text{m}$).

FIG. 10. The structural graphs of the density function of retail stores and restaurants, 1990 and 98 ($R=100\text{m}$). For the sake of simplicity, bottoms, cols, and corridors connecting bottoms and cols are omitted.

The results shown in Table 3 and Figure 10 are quite different from those of the global analysis. The number of peaks fluctuates sharply from 1990 to 96. During 90-92 and 94-96 a large number of subcenters appeared mainly by $G+$, which implies the opening of new stores. In contrast, nineteen peaks disappeared from 1992 to 94, nine by $D+$ and ten by $D-$. Disappearance of peaks, that is, integration of subclusters, occurred by both expansion and declination of existing clusters.

We then analyze the topological events in more detail in conjunction with development projects carried out in this region.

1991	4	New Tokyo Metropolitan Government Office (office building)
1994	7	Shinjuku Park Tower (office building)
1994	10	Ebisu Garden Place (office buildings and commercial facilities)
1995	1	Shinjuku Island Tower (office building)
1996	10	Shinjuku Takashimaya (department store)

Figure 11 shows that the openings of new office buildings have similar effect on the spatial structure of the retail cluster. From 1990 to 92, two peaks appeared around Tokyo Metropolitan Government Office, which reflects an increase in demand of employees for

restaurants and retail stores. Similarly, during 94-96, two peaks were newly generated near Shinjuku Park Tower and Shinjuku Island Tower. These three buildings are all very high office buildings having 48, 52, and 44 floors. From this we can say that the opening of very high office buildings greatly affects the spatial structure of retailing.

FIG. 11. Topological events of peaks, 90-98 ($R=100\text{m}$).

Effect of the opening of Ebisu Garden Place was more drastic; it had totally changed the spatial structure of retailing around Ebisu station as shown in Figures 10 and 11. From 1992 to 94 seven peaks disappeared; as a result, eleven peaks were integrated into only four peaks. Note that this change does not imply the decline of retailing because four out of seven disappearances are classified into D+; the number of stores had not necessarily decreased. We should rather say that rapid growth of retailing had promoted integration of subclusters in this region. On the other hand, after the opening of Ebisu Garden Place in 1994, six peaks were newly generated by G+. However, the location of peaks is quite different from that of the peaks that had disappeared before 1994. Consequently, we can conclude that Ebisu Garden Place had not only contributed to retail activity around Ebisu station but also greatly changed the spatial structure of retailing in that region.

In contrast to the above four projects, the opening of Shinjuku Takashimaya shows no significant effect in retail structure. We can find one generation (G+) at the south of Takashimaya, but it does not seem to be the result of the opening of Takashimaya. The effect of the opening of department stores may appear slowly compared to that of office buildings.

We finally examine local movement of peaks. Figure 12 clearly shows regional variation of movement pattern; peaks were continuously moving around Shinjuku, Shibuya, and Ebisu stations while rather stable around Yoyogi and Harajuku. This suggests that large retail clusters change their spatial structure more frequently than small clusters. We also notice, in contrast to global analysis, that peaks had not necessarily moved away from railway stations. Peaks had moved in various directions at the local scale without showing any clear tendency.

FIG. 12. Movement of peaks, 90-98 ($R=100\text{m}$).

4. CONCLUSION

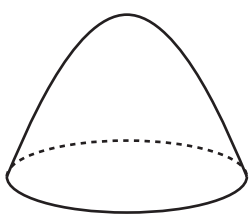
In this paper we have developed a method for analyzing the change of surfaces, extending the topological method to spatio-temporal domain. The spatial structure of a

surface is represented by its structural graph that consists of critical points and corridors. The change of a surface is described by a set of primitive events; generation, disappearance, and movement are considered for critical points, and switch is additionally introduced for corridors. From surfaces of two times a set of primitive events that causes the change is deduced. They are stored in GIS as spatial objects with attributes, and their spatio-temporal pattern is visually analyzed. To test the validity of the method, we analyzed the change of a retail cluster in Shinjuku and Shibuya area in Tokyo, Japan. The empirical study yielded some interesting findings that help us understand the change of the spatial structure of retailing.

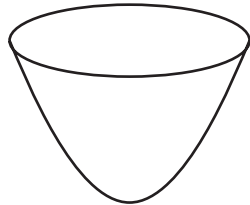
We finally discuss some limitations of our method for further research. First, we adopt a graph-theoretical representation of surfaces to describe their change. Primitive events are defined for the elements of the structural graph: peaks, bottoms, cols, and corridors. Though they typically characterize the global structure of surfaces, some applications may require more detailed description of surfaces. For example, one may be interested in the direction of slope, its spatial distribution, and its change over time. To deal with more details of surfaces, representation of surfaces and their change has to be extended to metric domain. Second, we interpolate surfaces between discrete times by linear interpolation, assuming linear change of surfaces over time. This assumption, however, may not be always valid. For example, let us consider a moving peak and its snapshot data. From the data we may deduce the change between two times as a combination of generation and disappearance, not movement. This misinterpretation can be avoided by acquiring spatial data with high frequency as we did in the empirical analysis. However, it is certainly desirable to adopt a spatio-temporal interpolation method suitable for the surface of analyst's interest. New spatio-temporal interpolation methods that explicitly take the behavior of surfaces into account should be developed in future research.

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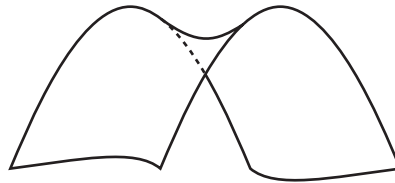
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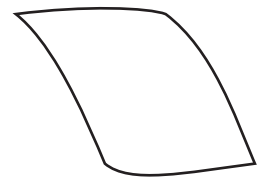
peak



bottom

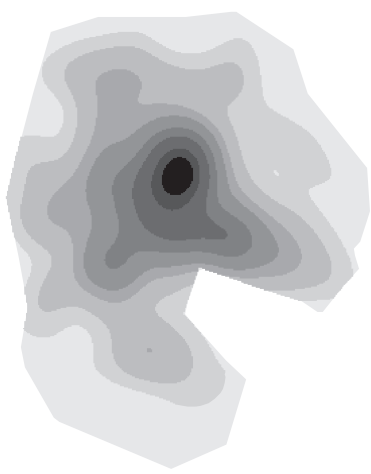


col

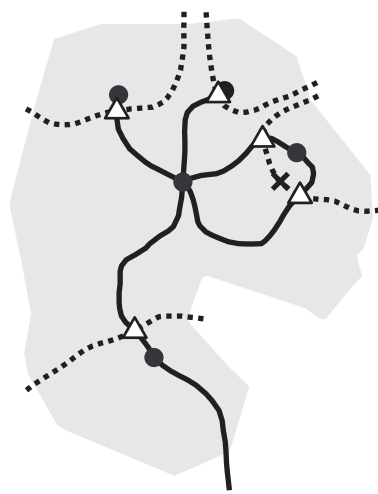


slope

Figure 1



(a)



(b)

- peak
- × bottom
- △ col
- corridor connecting a peak and a col
- ⋯ corridor connecting a bottom and a col

Figure 2

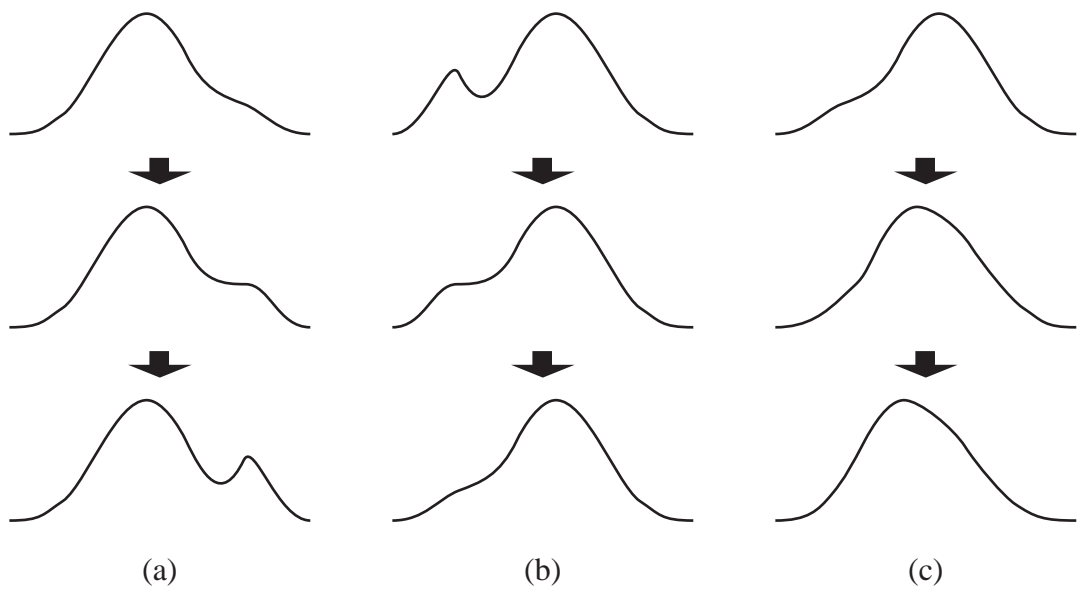


Figure 3

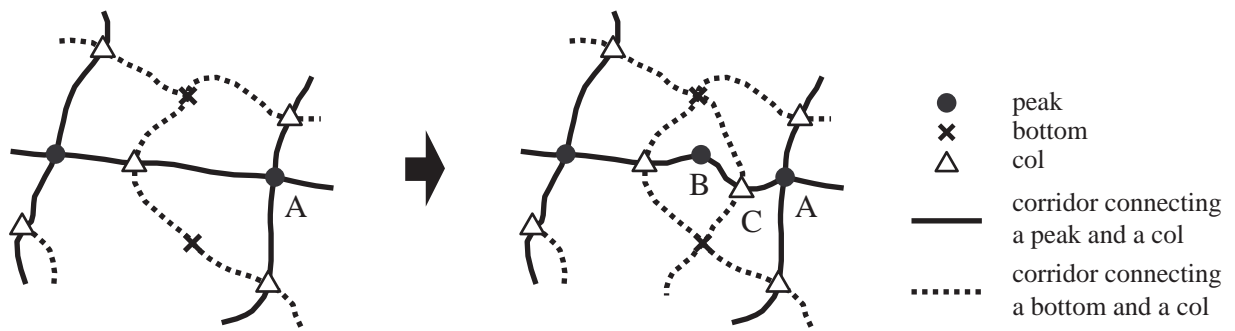


Figure 4

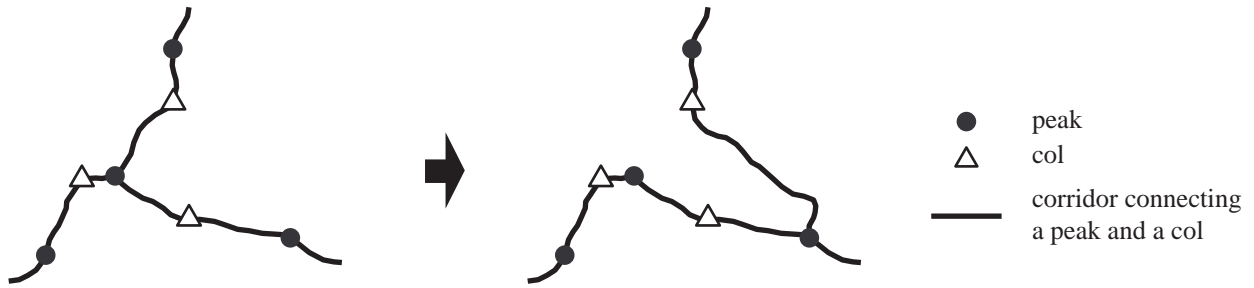


Figure 5

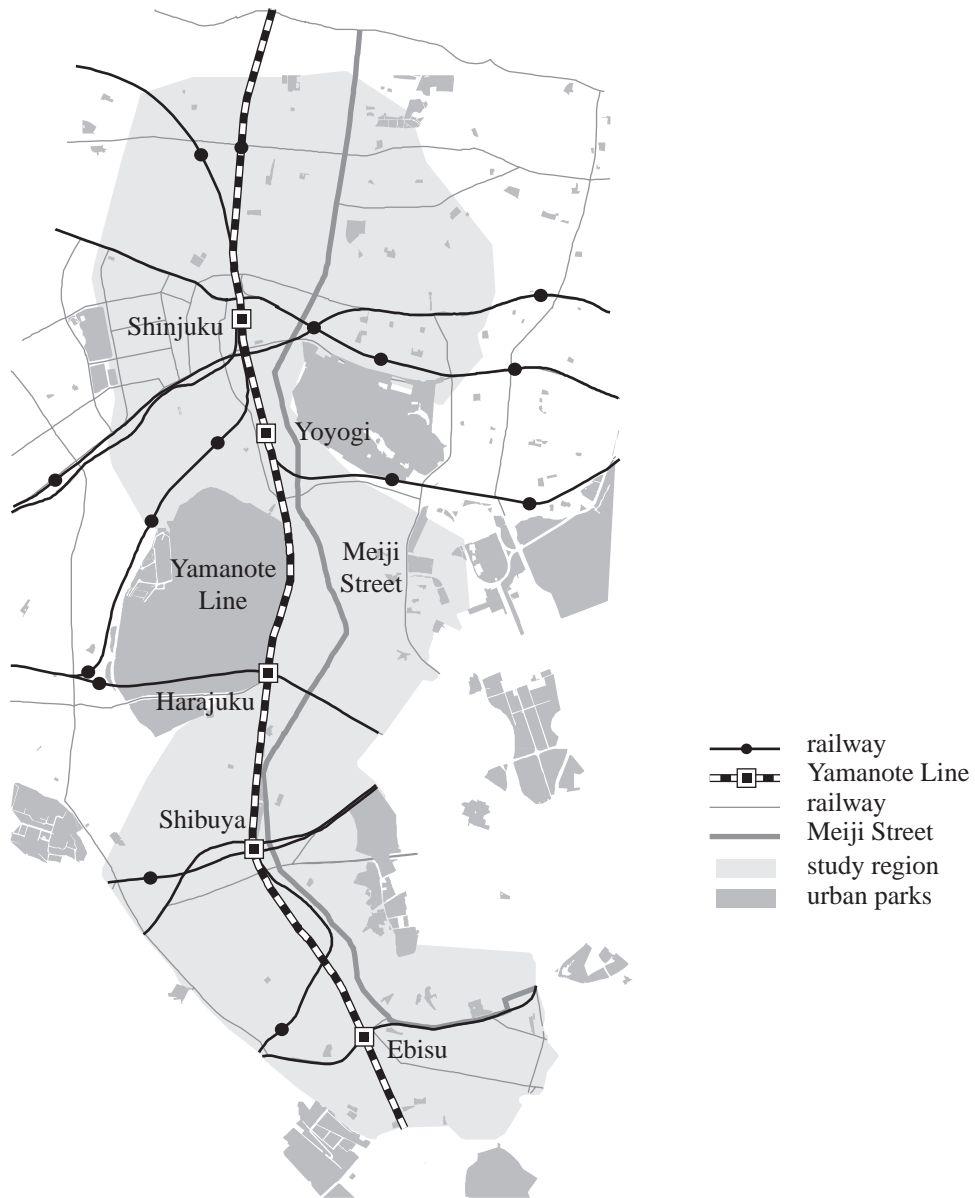


Figure 6

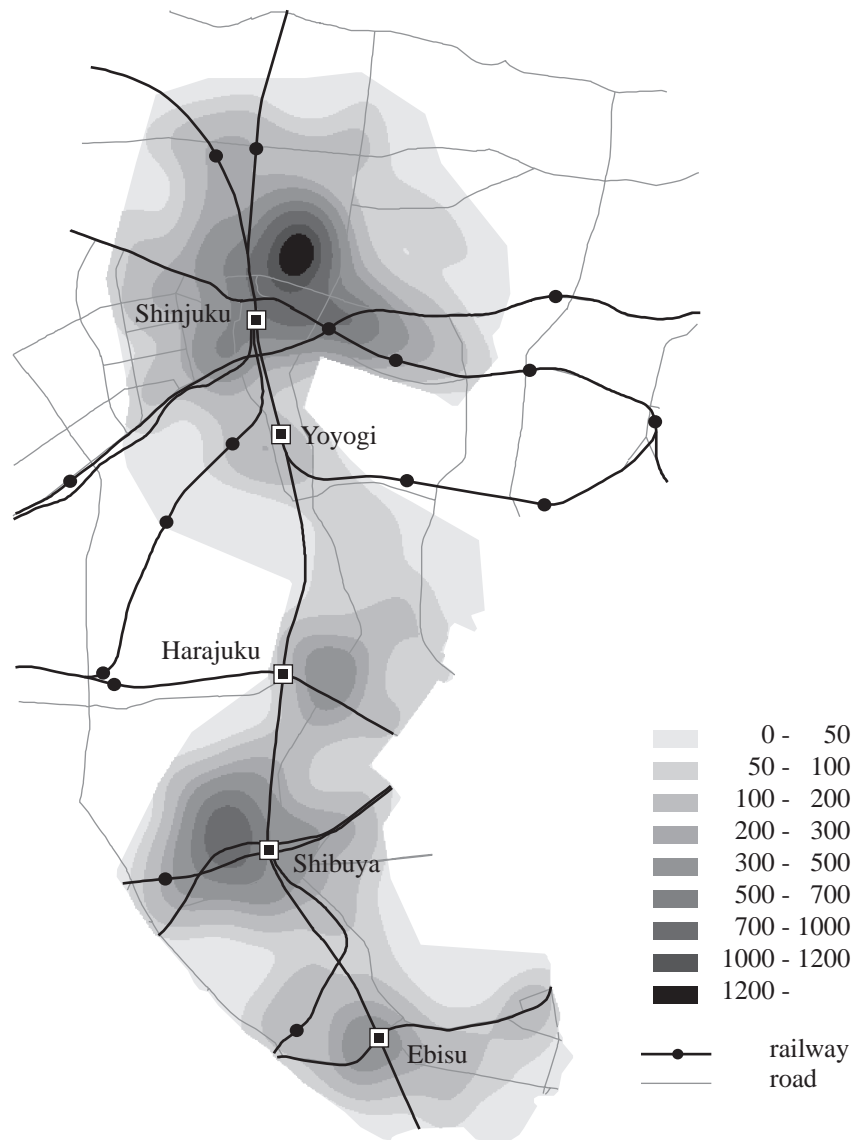


Figure 7

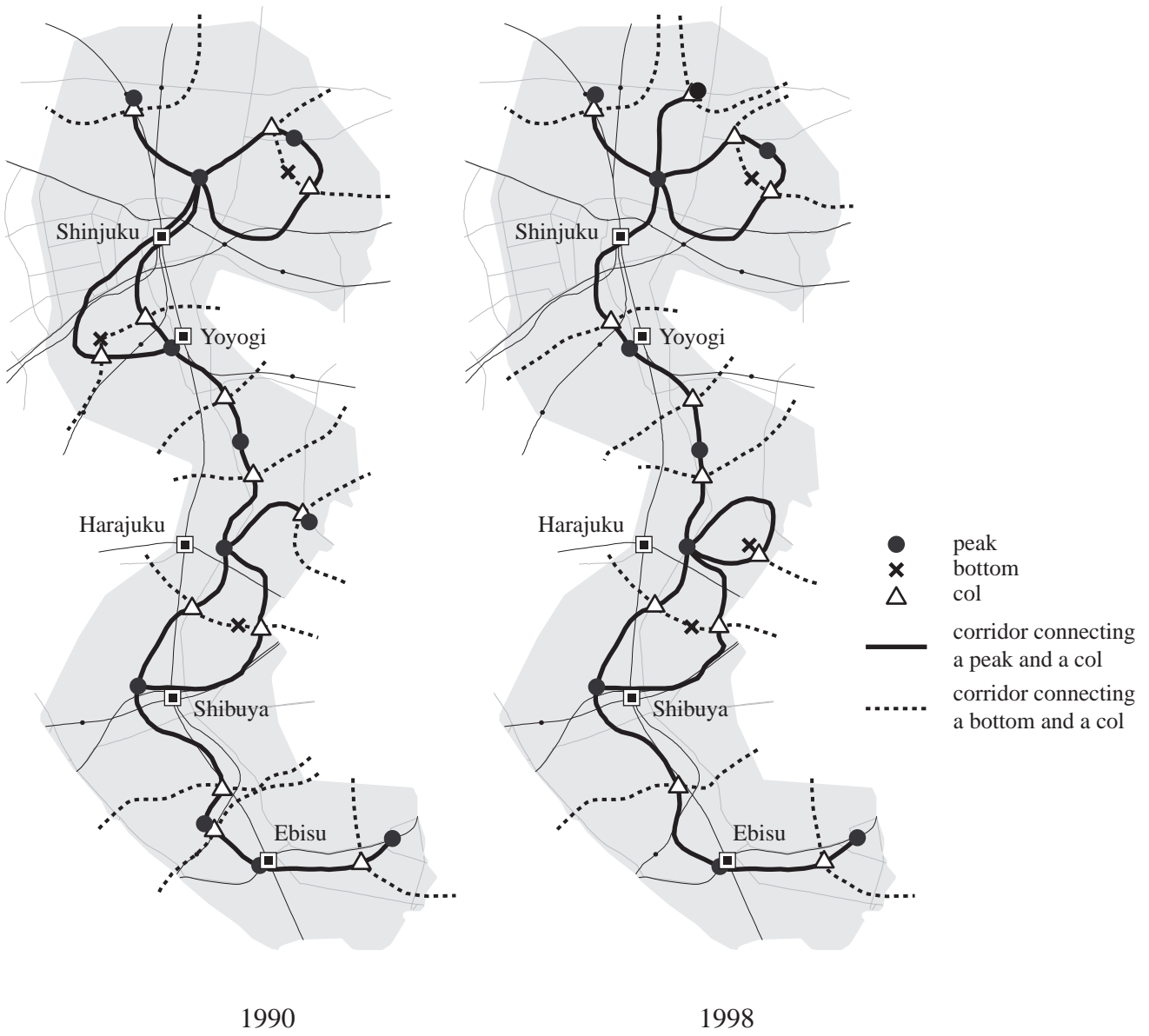


Figure 8

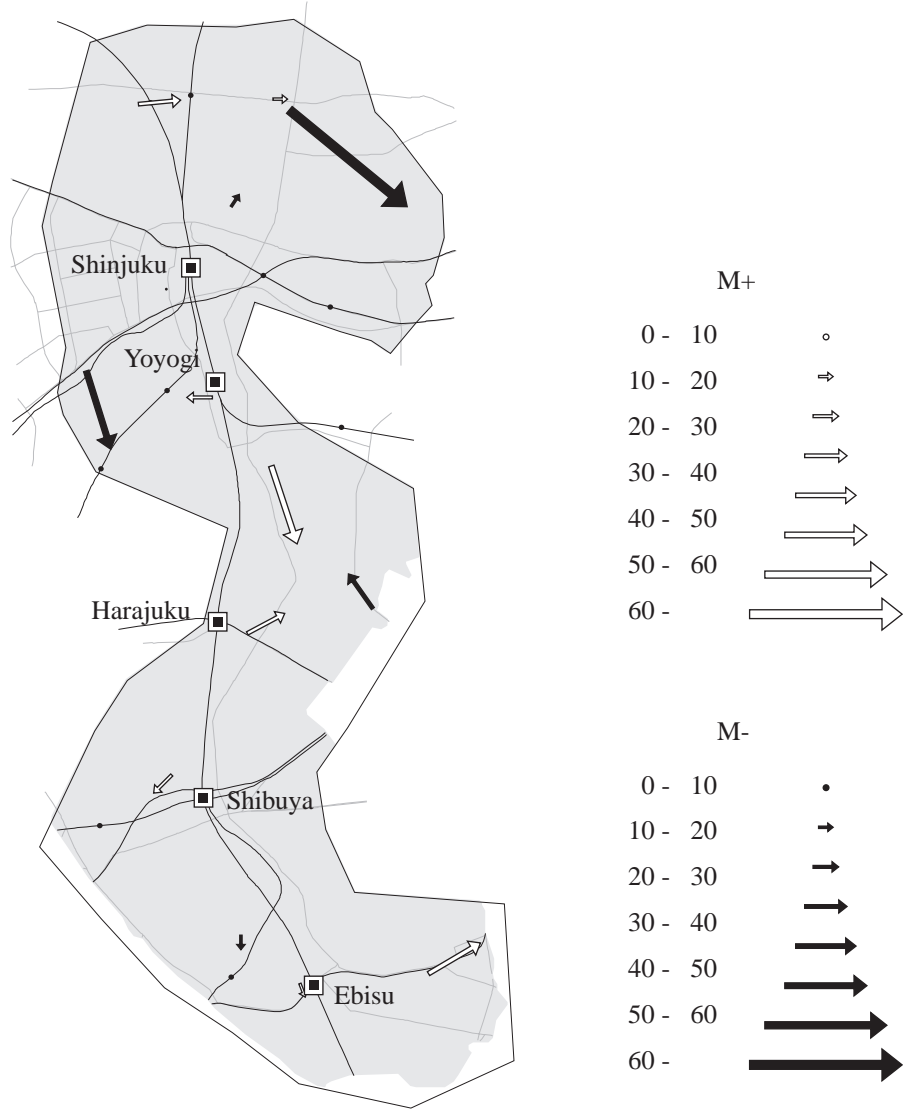


Figure 9

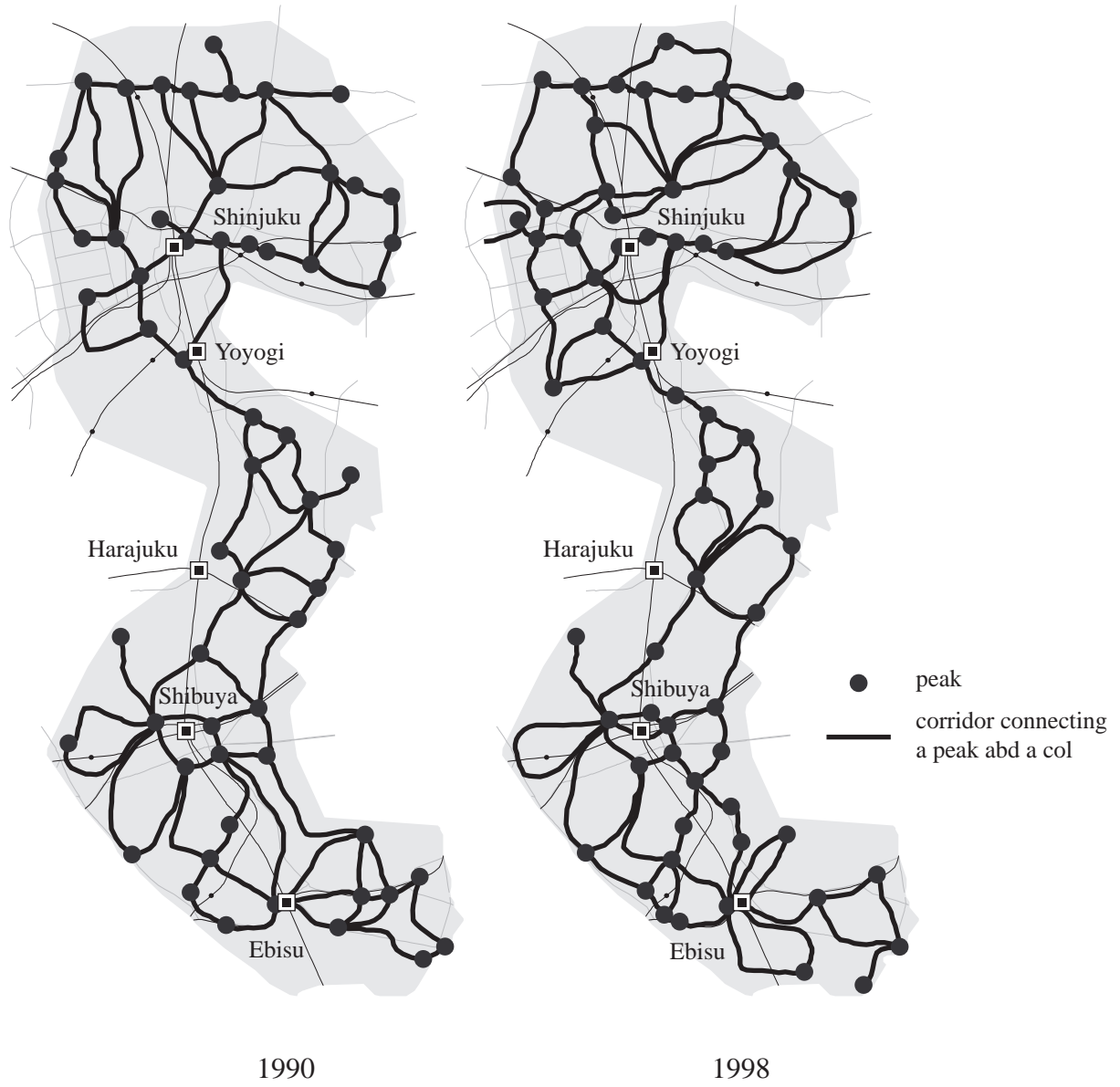


Figure 10

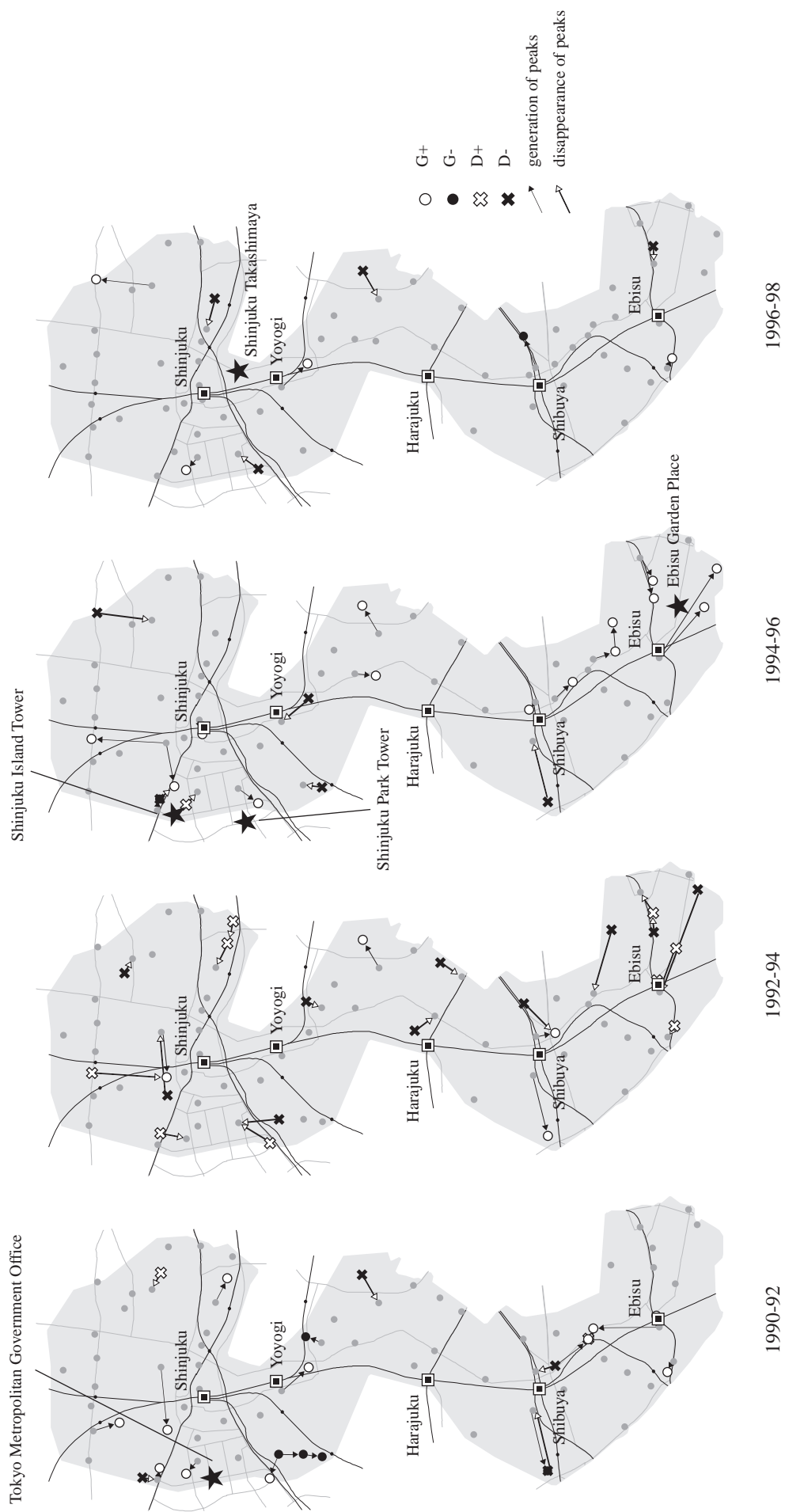


Figure 11

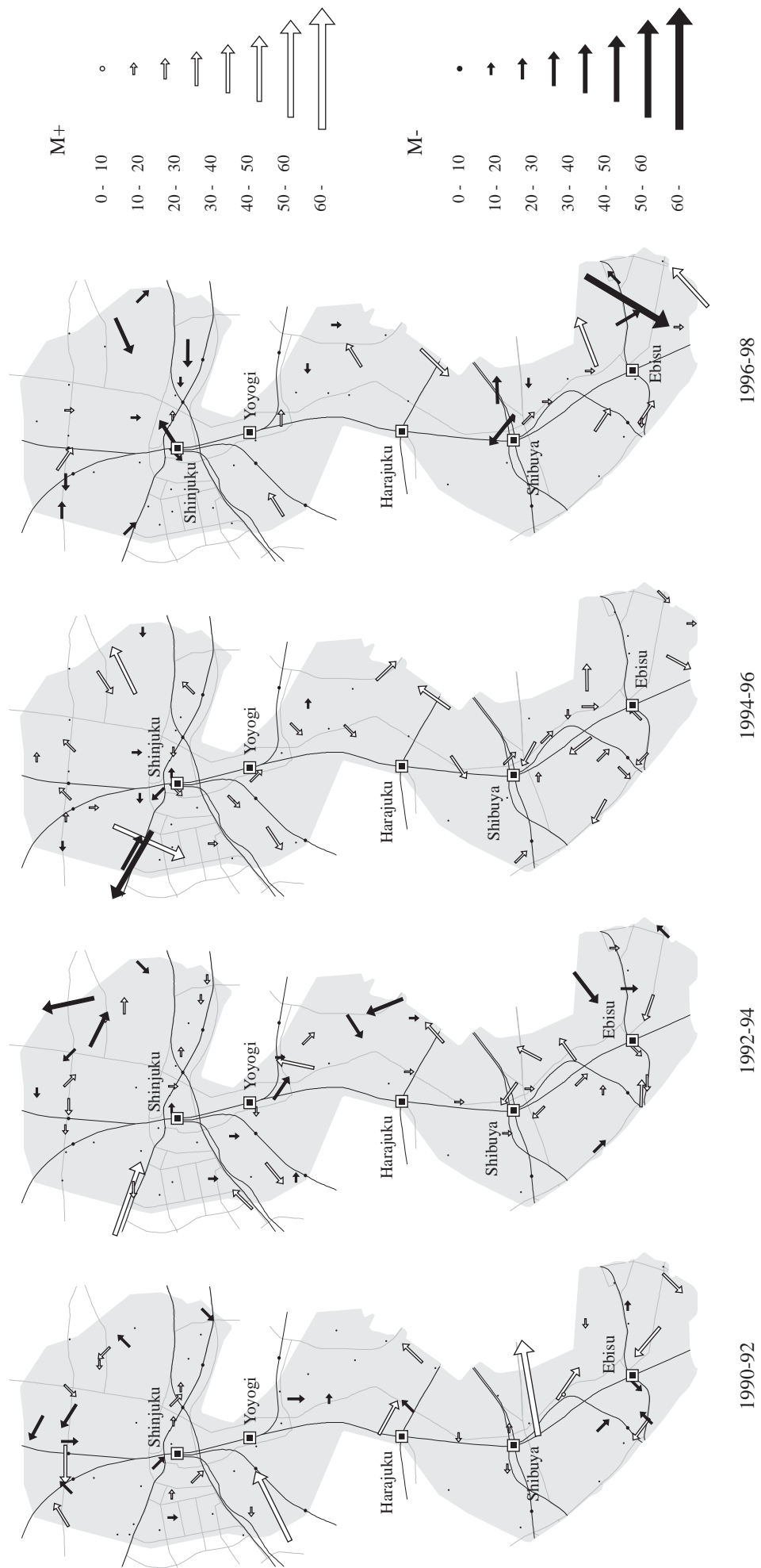


Figure 12

year	number of stores
1990	17176
1992	17486
1994	17960
1996	18520
1998	18797

Table 1

		1990	1992	1994	1996	1998
number of peaks		11	9	10	10	10
<hr/>						
		90-92	92-94	94-96	96-98	
	G+	1	1	1	0	
number of events	G-	1	0	0	1	
	D+	1	0	1	0	
	D-	1	0	0	1	

Table 2

		1990	1992	1994	1996	1998
number of peaks		60	70	55	63	64
<hr/>						
		90-92	92-94	94-96	96-98	
	G+	11	4	14	4	
number of events	G-	5	0	0	1	
	D+	2	9	1	0	
	D-	4	10	5	4	

Table 3